## BI

| Written examination: | GRA 60353 Mathematics |
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| Examination date: | $06.02 .2012 \quad$ 18:00-21:00 Total no. of pages: 1 |
| Permitted examination | A bilingual dictionary and BI-approved calculator TEXAS |
| support material: | INSTRUMENTS BA II Plus |

## Question 1.

We consider the function $f$ given by $f(x, y, z)=e^{x^{2}-y}+y+z^{2}$.
(a) Find all stationary points of $f$.
(b) Is $f$ convex? Is it concave?

Question 2.

We consider the matrix $A$ and the vector $\mathbf{v}$ given by

$$
A=\left(\begin{array}{ccc}
1 & 3 s+1 & -2 \\
3 & 7 s-2 & 0 \\
2 & 7 s & -4
\end{array}\right), \quad \mathbf{v}=\left(\begin{array}{c}
-8 \\
2 \\
3
\end{array}\right)
$$

(a) Compute the determinant and the rank of $A$.
(b) Is $\mathbf{v}$ an eigenvector for $A$ for any value of $s$ ? If so, what is the corresponding eigenvalue?
(c) Find all eigenvalues of $A$ when $s=2$.

## Question 3.

(a) Find the general solution $y=y(t)$ of the differential equation $y^{\prime \prime}+3 y^{\prime}-10 y=2 t$.
(b) Find the general solution $y=y(t)$ of the differential equation $2 t+3 y^{2} y^{\prime}=2 t e^{t^{2}}$.
(c) Find the solution $y=y(t)$ of the differential equation $y^{2}+2 t y \cdot y^{\prime}=1$ that satisfies $y(1)=3$.

## Question 4.

We consider the optimization problem

$$
\min x^{2}+y^{2}+z^{2} \text { subject to } 2 x^{2}+6 y^{2}+3 z^{2} \geq 36
$$

(a) Write down the first order conditions and the complementary slackness conditions for the optimization problem. Find all admissible points that satisfy the first order conditions and the complementary slackness conditions, and write down the value of $\lambda$ for each of these points.
(b) Solve the optimization problem and compute the minimum value. Give an argument that proves that your solution is a minimum.

