Note: How to compute all principal minors

A: symmetric $n \times n$-matrix

D_i = minor obtained by keeping $\begin{cases} \text{row} & 1,2,3,\ldots,i \\ \text{col} & 1,2,3,\ldots,i \end{cases}$ (the ~~first~~ i rows and cols) for $i = 1,2,\ldots,n$. } leading principal minors

Δ_i = a minor obtained by keeping $\{$ i rows, the same i cols $\}$ (need not be the first rows and cols, but must be the same rows and cols) } all principal minors

Ex:

$$A = \begin{pmatrix} a & 1 & 0 & 0 \\ 1 & a & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Mock exam Dec 2013 Problem 2

D_i:

$$\begin{pmatrix} \underline{a} & \underline{1} & 0 & 0 \\ \underline{1} & \underline{a} & 0 & 0 \\ 0 & 0 & \underline{1} & \underline{-1} \\ 0 & 0 & \underline{-1} & \underline{1} \end{pmatrix}$$

$D_1 = \underline{a}$ $\quad D_2 = \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} = \underline{a^2 - 1}$, $\quad D_3 = \begin{vmatrix} a & 1 & 0 \\ 1 & a & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot (a^2 - 1) = \underline{a^2 - 1}$ $\quad D_4 = |A| = 0$ (see 2a)

Δ_i: (A1) Choose row / col: 1 2 3 4 / 1 2 3 4

$$A = \begin{pmatrix} a & 1 & 0 & 0 \\ 1 & a & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$\Delta_1 = a \quad a \quad 1 \quad 1$

$\uparrow D_1$

(A2) Choose rows / cols: 1,2 1,3 1,4 2,3 2,4 3,4 / 1,2 1,3 1,4 2,3 2,4 3,4

$\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} \quad \begin{vmatrix} a & 0 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} a & 0 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} a & 0 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} a & 0 \\ 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$

$= a^2 - 1 \quad = a \quad = a \quad = a \quad = a \quad = 0$

$\uparrow D_2$

$$A = \begin{pmatrix} a & 1 & 0 & 0 \\ 1 & a & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

+ three where you shift rows/cols

(A3) Choose rows / cols: 1,2,3 1,2,4 1,3,4 2,3,4 / 1,2,3 1,2,4 1,3,4 2,3,4

$\begin{vmatrix} a & 1 & 0 \\ 1 & a & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \begin{vmatrix} a & 1 & 0 \\ 1 & a & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \begin{vmatrix} a & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{vmatrix}$

$= a^2 - 1 \quad = a^2 - 1 \quad = a \cdot 0 = 0$

$\uparrow D_3$

$\begin{vmatrix} a & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{vmatrix}$

$= a \cdot 0 = 0$

(A4) $= D_4 = 0$

(only one possible choice)