## Key Problems

**Problem 1.**
We consider the vectors $\mathbf{v}_1 = (1,3,4),\ \mathbf{v}_2 = (-1,3,4),\ \mathbf{v}_3 = (5,3,4),\ \mathbf{v}_4 = (6,4,5),\ \mathbf{v}_5 = (4,2,3)$.

a) Is $\mathbf{v}_3$ in $\text{span}(\mathbf{v}_1, \mathbf{v}_2)$?

b) Express $\mathbf{v}_5$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ if possible.

c) Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$ linearly independent vectors?

d) Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5$ linearly independent vectors?

e) Determine the dimension of $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5)$, and find a base of $V$.

f) Express $\mathbf{v}_5$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in a different way than in (b), if possible.

**Problem 2.**
Find a parametric description of the line through the points (1,2,1) and (4,5,3) in $\mathbb{R}^3$. Determine the intersection points $(x,y,z)$ of this line and the plane $x - y + z = 6$.

**Problem 3.**
Determine $\dim V$ and $\dim W$ when $V = \text{Col}(A)$, $W = \text{Null}(A)$, and $A$ is the $3 \times 5$ matrix $A$ given below, and find a base of $V$ and $W$:

$$A = \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 2 & 4 & -2 & -2 & -2 \\ 3 & 5 & -1 & -1 & -1 \end{pmatrix}$$

**Problem 4.**
Let $A$ be a $8 \times 8$ matrix with rank given by $\text{rk}(A) = 7$ and let $\mathbf{b}$ be a vector in $\mathbb{R}^8$. Determine:

a) $\dim \text{Null}(A)$ and $\dim \text{Col}(A)$

b) The number of solutions of $A\mathbf{x} = \mathbf{0}$

c) The number of solutions of $A\mathbf{x} = \mathbf{b}$

d) The number of solutions of $A\mathbf{x} = \mathbf{0}$ that satisfies $x_1 + x_2 + \cdots + x_8 = 1$

**Problem 5.**
We consider the vectors $\mathbf{u} = (1,1,2,1)$, $\mathbf{v} = (2,4,-1,2)$ and $\mathbf{w} = (1,2,4,2)$.

a) Compute $\mathbf{u} \cdot \mathbf{v}$ and $||\mathbf{u}||$.

b) Find the orthogonal projection $\text{proj}_{\mathbf{v}}(\mathbf{u})$ of $\mathbf{u}$ onto $\mathbf{v}$.

c) Determine the scalar $a$ such that $\mathbf{v} - a \cdot \mathbf{w}$ is orthogonal to $\mathbf{w}$.

## Exercise problems

| | |
|---|---|
| Problems from the textbook: | [E] 2.1 - 2.16 |
| Exam problems: | [Midterm 10/2019] Question 1, 2, 8 |
| | [Midterm 10/2022] Question 1, 2, 7 |

Eivind Eriksen, Office B3y-085, eivind.eriksen@bi.no
https://www.dr-eriksen.no/teaching/GRA6035/
https://www.dr-eriksen.no/teaching/ELE3781/
BI Norwegian Business School
MSc Business/Finance
BSc Data Science

# Answers to Key Problems

**Problem 1.**

a) Yes

b) $\mathbf{v}_5 = 6\mathbf{v}_1 - 4\mathbf{v}_2 - \mathbf{v}_4$

c) Yes

d) Yes

e) $\dim V = 3$, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of $V$

f) $\mathbf{v}_5 = 2\mathbf{v}_3 - \mathbf{v}_4$

**Problem 2.**

Parametric description: $(x,y,z) = (1+3t, 2+3t, 1+2t)$. Intersection point: $(x,y,z) = (10,11,7)$.

**Problem 3.**

a) $\dim V = 3$, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of $V$ when $\mathbf{v}_i$ is the $i$'th column vectors of $A$

b) $\dim W = 2$, and $\{\mathbf{w}_1, \mathbf{w}_2\}$ is a base for $W$ when $\mathbf{w}_1 = (-3,2,1,0,0)$, $\mathbf{w}_2 = (-6,4,0,1,1)$

**Problem 4.**

a) $\dim \text{Null}(A) = 1$ and $\dim \text{Col}(A) = 7$

b) Infinitely many solutions (one degree of freedom)

c) Infinitely many solutions (one degree of freedom) if $\mathbf{b}$ is a linear combination of the columns of $A$, otherwise no solutions

d) No solutions if $(1,1,\ldots,1)$ is a linear combination of the rows of $A$, otherwise one unique solution.

**Problem 5.**

a) $\mathbf{u} \cdot \mathbf{v} = 6$, $||\mathbf{u}|| = \sqrt{7}$

b) $\text{proj}_{\mathbf{v}}(\mathbf{u}) = {}^{6}\!/_{25} \cdot \mathbf{v}$

c) $a = 2/5$