

Key Problems

You should do the Key Problems as soon as possible after each lecture. They test your understanding of the most important theories and methods presented in the lecture. You can work on Key Problems and get help in the Exercise session/TA session the following Monday (Campus Bergen: Wednesday).

Problem 1.

Solve the linear system $A\mathbf{x} = \mathbf{b}$ using Gaussian elimination:

$$\text{a) } A = \begin{pmatrix} 1 & 3 & 4 \\ 5 & 1 & 8 \\ 4 & 5 & 9 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 11 \\ 15 \\ 23 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 4 & 5 & 3 & 11 \\ 2 & 5 & 0 & 3 \\ 3 & 2 & 3 & 7 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 23 \\ 17 \\ 14 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 1 & 1 & 1 & 1 & 4 \\ 1 & 3 & 1 & 5 & 18 \\ 2 & 4 & 2 & 9 & 31 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 8 \\ 28 \\ 48 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 4 & 1 \\ 7 & 7 & 7 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 13 \\ 11 \\ 17 \end{pmatrix}$$

Problem 2.

Find the rank of the following matrix:

$$A = \begin{pmatrix} 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 1 \\ 3 & 0 & 2 & 3 & 1 \\ 3 & 6 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & -1 \end{pmatrix}$$

Problem 3.

Let V be the set of solutions of the linear system $A\mathbf{x} = \mathbf{b}$ from Problem 1(b). What is the dimension of V ? Describe the set V geometrically.

Problem 4.

We consider the following linear system. Find all solutions that satisfies $x + w = y + z$:

$$\begin{aligned} x + y + 2z + 4w &= 6 \\ x + 2y + 4z - 2w &= 9 \\ x + 3y + 9z + 7w &= 24 \end{aligned}$$

Problem 5.

We consider a 4×5 homogeneous linear system $A\mathbf{x} = \mathbf{0}$, and $\text{rk } A = 4$. How many solutions are there?

Exercise Problems

In addition to the Key Problems, you should try to do as many of the Exercise Problems as you can. They are usually problems from the textbook or exam problems. You can work with Exercise Problems and get help in the Exercise sessions/TA sessions on Mondays (Wednesdays). If you do not have time for all Exercise Problems in the following week, make sure to do them later in the semester as revision.

Problems from the textbook: [E] 1.1 - 1.17

Answers to Key Problems

Problem 1.

a) $(1,2,1)$

c) $(2 - s, 2 - t, s, 4 - 3t, t)$

b) $(6 + 11t, 1 - 5t, -2 - 10t, t)$

d) No solutions

Problem 2.

$$\text{rk}(A) = 5$$

Problem 3.

We have that $\dim V = 1$ since there is one degree of freedom. The solution set V is a straight line in four-dimensional space \mathbb{R}^4 .

Problem 4.

$$(x, y, z, w) = (1, -9/5, 3, 1/5)$$

Problem 5.

Infinitely many solutions (one degree of freedom).

Answer to Exercise Problems

You find full solutions to all problems in the textbook [E] in the workbook [EP] Eriksen, *Graduate mathematics for Business, Economics and Finance - Problems and Solutions*, and full solutions to all exam problems on the course webpage.