

GRA 6035 MATHEMATICS

Problems for Lecture 6

Key problems

Problem 1.

Determine the definiteness of the quadratic form f :

- a) $f(x, y, z) = 5x^2 + 6xy + 2y^2 + 16xz + 10yz + 13z^2$ b) $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 + 2xz - 2yw$
c) $f(x, y, z, w) = 2xy + 2xz + 2yw + 2zw$ d) $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$

Problem 2.

Determine all values of a such that the symmetric matrix A is negative semidefinite:

$$A = \begin{pmatrix} a & 0 & 0 & -1 \\ 0 & a & -1 & 0 \\ 0 & -1 & a & 0 \\ -1 & 0 & 0 & a \end{pmatrix}$$

Problem 3.

Find all stationary points of f , classify them as local maximum/minimum points or saddle points, and determine whether f has global maximum/minimum values:

- a) $f(x, y, z) = xy + xz - yz$ b) $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$ c) $f(x, y, z) = x^4 + y^4 + z^4 + z^2$
d) $f(x, y, z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$

Problem 4.

Determine whether f is a convex or concave function:

- a) $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 + xy + yz + zw$ b) $f(x, y, z) = e^{x-2y+z}$ c) $f(x, y, z) = x^4 + y^4 + z^4 + z^2$
d) $f(x, y, z) = 16 - x^4 - 2x^2 - 3y^2 + 6xz - 6z^2$ e) $f(x, y, z) = (xy + xz + yz)/xyz$ defined for $x, y, z > 0$

Problems from the Digital Workbook

Exercise problems	6.1 - 6.20 (full solutions in the workbook)
Exam problems	6.21 - 6.26 (full solutions in the workbook)
	Midterm exam 01/2018 Question 7, Midterm exam 05/2018 Question 7
	Midterm exam 10/2017 Question 1-8

Answers to key problems

Problem 1.

- a) Positive semi-definite b) Positive semi-definite c) Indefinite d) Positive definite

Problem 2.

It is negative semi-definite for $a \leq -1$

Problem 3.

- a) Saddle point $(0, 0, 0)$, no global max/min value b) Local min $(0, 0, 0, 0)$, global min value $f_{\min} = 0$, no global max value
c) Local min $(0, 0, 0)$, global min value $f_{\min} = 0$, no global max value d) Local max $(0, 0, 0)$, global max value $f_{\max} = 16$, no global min value

Problem 4.

- a) Convex b) Convex c) Convex d) Concave e) Convex