GRA 6035 Mathematics

Problems for Lecture 3

Key problems

Consider the 3-vectors given by

$$\mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 0 \\ 9 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}, \quad \mathbf{v}_5 = \begin{pmatrix} 4 \\ 3 \\ 9 \end{pmatrix} \quad \text{and} \quad \mathbf{w} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Problem 1.
In each case, determine when $\mathbf{w}$ is in the span V, and compute the dimension of V:
a) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2)$ b) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ c) $V = \text{span}(\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$ d) $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$

Problem 2.
For each set of vectors, determine if the vectors are linearly independent:
a) $\{\mathbf{v}_1, \mathbf{v}_2\}$ b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ d) $\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ e) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

Problem 3.
Find $\text{Null}(A)$ for the matrix $A = (\mathbf{v}_1|\mathbf{v}_2|\mathbf{v}_3|\mathbf{v}_4|\mathbf{v}_5)$; that is, the set of solutions of the homogeneous linear system $A \cdot \mathbf{x} = \mathbf{0}$. Write $\text{Null}(A)$ as the span of a set $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_r\}$ of linearly independent vectors. What is the value of r?

Problems from the Digital Workbook

Exercise problems	3.1 - 3.12 (full solutions in the workbook)
Exam problems	3.13 - 3.15 (full solutions in the workbook)

Answers to key problems

Problem 1.
a) When $6a - b - 2c = 0$, and $\dim V = 2$ b) For all a, b, c, and $\dim V = 3$ c) When $b + c - 3a = 0$, and $\dim V = 2$
d) For all a, b, c, and $\dim V = 3$

Problem 2.
a) Yes b) Yes c) Yes d) No e) No

Problem 3.
We have that $\text{Null}(A) = \text{span}(\mathbf{w}_1, \mathbf{w}_2)$ with $r = 2$ and

$$\mathbf{w}_1 = \begin{pmatrix} 0 \\ -5 \\ 3 \\ 6 \\ 0 \end{pmatrix}, \quad \mathbf{w}_2 = \begin{pmatrix} 0 \\ -5 \\ -9 \\ 0 \\ 6 \end{pmatrix}$$