PLENARY SESSION 2

GRA 6035

MATHEMATICS

EIVIND ERIKSEN SEP 28, 2015

Problems:

Lecture 4: 4.4, 4.6 – 4.12

Lecture 5: 5.4 – 5.5, 5.8 – 5.9, 5.13

Lecture 6: $_\wedge$ 6.2 ii, 6.4 iii, 6.5, 6.8, 6.20, 6.26

6.1 iii)

4.4 A is invertible, λ eigenvalue

Since $|A| = \textcircled{\lambda} \lambda_2 \lambda_3 \ldots \lambda_n \neq 0$, $\underline{\lambda \neq 0}$

Moreover:

$A \cdot \underline{v} = \lambda \underline{v}$ for each $\underline{v}$ in $E_\lambda(A)$

$A^{-1} \cdot A \underline{v} = A^{-1} \cdot \lambda \underline{v} \quad | \; A^{-1} \text{ fro}$

$\underline{v} = \lambda \cdot A^{-1} \underline{v} \quad | \; \frac{1}{\lambda} \cdot$

$\frac{1}{\lambda} \underline{v} = A^{-1} \underline{v} \longrightarrow$ $\underline{v}$ is eigenvector for A^{-1} with eigenvalue $1/\lambda = \lambda^{-1}$

4.6 $A = \begin{pmatrix} 2 & -7 \\ 3 & -8 \end{pmatrix}$ $\quad \lambda^2 + 6\lambda + 5 = 0$

$\underline{\lambda = -1}, \; \underline{\lambda = -5}$

$D = \begin{pmatrix} -1 & 0 \\ 0 & -5 \end{pmatrix}$ $\quad P = \left(\underline{v_1} \mid \underline{v_2} \right) = \underline{\underline{\begin{pmatrix} 7 & 1 \\ 3 & 1 \end{pmatrix}}}$

$\underline{\lambda = -1}$: $\begin{pmatrix} 3 & -7 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ $\quad \underline{v_1} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$\underline{\lambda = -5}$: $\begin{pmatrix} 7 & -7 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad \underline{v_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

4.7 $A = \begin{pmatrix} 3 & 5 \\ 0 & 3 \end{pmatrix}$

$$\begin{vmatrix} 3-\lambda & 5 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\cdot(3-\lambda) = 0$$

Eigenvalue: $\underline{\lambda_1 = \lambda_2 = 3}$

$$D_1 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \quad \underline{V}$$

Eigenvectors:

$$P = \left(\begin{matrix} 1 \\ 0 \end{matrix} \,\middle|\, \underset{\text{missing}}{\uparrow} \right)$$

$\underline{\lambda = 3}$: $\begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\left. \begin{matrix} 5y = 0 \\ y = 0 \end{matrix} \right\} \quad \underline{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

one degree of freedom

$\Downarrow$

A $\underline{\underline{\text{not}}}$ diagonalizable $\Longleftarrow$ not enough eigenvectors

4.8 $T = \begin{pmatrix} 0.85 & 0.45 \\ 0.15 & 0.55 \end{pmatrix}$ $\underline{s} = \begin{pmatrix} 0.2 \\ 0.8 \end{pmatrix}$

a) $T \cdot \underline{s} = \begin{pmatrix} 0.53 \\ 0.47 \end{pmatrix}$ marked share vector: $0{,}53\,,\ 0.47 \geq 0$ ⓥ

$0.53 + 0.47 = 1$

b) Eigenvalues:

$$\begin{vmatrix} 0.85-\lambda & 0.45 \\ 0.15 & 0.55-\lambda \end{vmatrix} = 0$$

$\underline{\begin{matrix} \lambda_1 = 1 \\ \lambda_2 = 0{,}40 \end{matrix}}$ eigenvalues

$$\lambda^2 - 1.40\lambda + 0.40 = 0$$

Eigenvectors:

$\underline{\lambda = 1}$: $\begin{pmatrix} -0{,}15 & 0.45 \\ 0{,}15 & -0.45 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \underline{t \begin{pmatrix} 3 \\ 1 \end{pmatrix}}$

$\underline{\lambda = 0{,}40}$: $\begin{pmatrix} 0.45 & 0.45 \\ 0{,}15 & 0.15 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \underline{t \begin{pmatrix} -1 \\ 1 \end{pmatrix}}$

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 0.40 \end{pmatrix} \qquad \underline{P = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}}$$

$$\boxed{TP = P \cdot D} \qquad T = \cancel{P^{-1}DP} \; PDP^{-1}$$

$$D = P^{-1}TP$$

$$T^n = (PDP^{-1})^n = (PD\cancel{P^{-1}})(\cancel{P}D\cancel{P^{-1}}) \ldots (\cancel{P}DP^{-1})$$

$$= \underline{PD^nP^{-1}}$$

c) $$D^n \Rightarrow \begin{pmatrix} 1^n & 0 \\ 0 & 0.40^n \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{as } n \to \infty$$

$$T^n \to P \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot P^{-1} = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \frac{1}{4} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} 3 & 0 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{pmatrix}}}$$

as $n \to \infty$

<u>Equilibrium</u>, $$\lim_{n \to \infty} T^n \cdot \underline{s} = \begin{pmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.8 \end{pmatrix}$$

$$= 0.2 \cdot \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix} + 0.8 \cdot \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$$

$$= 1 \cdot \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}}}$$

4.11.

$$A = \begin{pmatrix} 1 & 1 & -4 \\ 0 & t+2 & t-8 \\ 0 & -5 & 5 \end{pmatrix}$$

a) $|A| = 1\cdot(5(t+2)+5(t-8)) = \underline{10t-30} = 10(t-3)$

$\text{rk}\, A = 3$ if $t \neq 3$ since $|A| \neq 0$

If $t=3$:
$$\begin{pmatrix} ① & 1 & -4 \\ 0 & ⑤ & -5 \\ 0 & -8 & 5 \end{pmatrix} \Rightarrow \text{rk}\, A = 2 \quad \text{when } t=3$$

$$\text{rk}\, A = \begin{cases} 3, & t \neq 3 \\ 2, & t = 3 \end{cases}$$

b)
$$\begin{vmatrix} 1-\lambda & 1 & -4 \\ 0 & t+2-\lambda & t-8 \\ 0 & -5 & 5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\cdot\left(\lambda^2 - (t+7)\lambda + \underbrace{5(t+2)+5(t-8)}_{10t-30}\right) = 0$$

$\underline{\lambda_1 = 1}$

$$\lambda = \frac{(t+7) \pm \sqrt{(t+7)^2 - 4\cdot(10t-30)}}{2}$$

$$= \frac{t+7 \pm \sqrt{t^2+14t+49-40t+120}}{2}$$

$$= \frac{t+7 \pm \sqrt{t^2-26t+169}}{2}$$

$\lambda_2 = \underline{t-3}$

$\lambda_3 = \underline{10}$ ← $= \dfrac{t+7 \pm \sqrt{(t-13)^2}}{2} = \dfrac{t+7 \pm (t-13)}{2}$

c) $\lambda_1 = 1$ $\lambda_2 = t-3$ $\lambda_3 = 10$

Enough eigenvalues $\longrightarrow$ $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & t-3 & 0 \\ 0 & 0 & 10 \end{pmatrix}$

$\underline{t \neq 4, 13:}$ $\underline{\text{Diagonalizable}}$

$\underline{t=4:}$ $\underline{\lambda = 1 \text{ has mult. } 2}$

$A-\lambda$ with $\lambda=1, t=4$ $\longrightarrow$ $\begin{pmatrix} 0 & \boxed{1} & -4 \\ 0 & 5 & \boxed{-4} \\ 0 & -5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

rk = 2

free vars =

3 − 2 = 1

not enough.

not diag.

$\underline{t=13:}$ $\underline{\lambda = 10 \text{ has mult } 2}$

$A-\lambda$ with $\lambda=10$ $t=13$ $\longrightarrow$ $\begin{pmatrix} \boxed{-9} & 1 & -4 \\ 0 & \boxed{5} & 5 \\ 0 & -5 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

rk = 2

free vars = 1

not enough

⇓

not diag.

Conclusion: $\begin{cases} A \text{ diag. when } t \neq 4, 13 \\ A \text{ \underline{not} diag. when } t=4,\ t=13 \end{cases}$

4.10 $A = \begin{pmatrix} 1 & 7 & -2 \\ 0 & s & 0 \\ 1 & 1 & 4 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

b) Is $\underline{v}$ eigenvector?

$$A \cdot \underline{v} = \begin{pmatrix} 1 & 7 & -2 \\ 0 & s & 0 \\ 1 & 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ s \\ 6 \end{pmatrix} = \lambda \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\underline{v}$ is a eigenvector if and only if $s=6$

(with $\lambda = 6$)

$6 = \lambda$
$s = \lambda$
$6 = \lambda$

5.4: $A = \begin{pmatrix} \textcircled{1} & 2 & 0 \\ 2 & \textcircled{4} & 5 \\ 0 & 5 & \textcircled{6} \end{pmatrix}$

leading princ.	prin. minors
$D_1 = 1$	$\Delta_1 = 1, 4, 6$
$D_2 = 0$	$\Delta_2 = 0, 6, -1$
$D_3 = 1(-1) - 2 \cdot 12$ $= -25$	$\Delta_3 = -25$

$Q = x_1^2 + 4x_1x_2 + 4x_2^2 + 10x_2x_3 + 6x_3^2$

indefinite $\Rightarrow$ $(0,0,0)$ is saddle pt

$\boxed{H(Q) = 2 \cdot A}$ for all quadratic forms

5.5. $\begin{pmatrix} a & b \\ b & \textcircled{c} \end{pmatrix}$

$D_1 = a$	$\Delta_1 = a, c$
$D_2 = ac - b^2$	$\Delta_2 = ac - b^2$

$a > 0, \; ac - b^2 > 0 \iff$ pos. defn.

$a < 0, \; ac - b^2 > 0$ neg. defn.

$a \geq 0, c \geq 0, \; ac - b^2 \geq 0 \iff$ pos. semidefn.

$a \leq 0, c \leq 0, \; ac - b^2 \geq 0$ neg. semidefn.

all other cases: indefinite

$ac - b^2 < 0$

BI

5.8. $$A = \begin{pmatrix} 0.87 & 0.16 \\ 0.13 & 0.84 \end{pmatrix}$$

regular Markov chain

$$\boxed{\begin{array}{l}\text{all entries} > 0 \\ \text{col. sums} = 1\end{array}}$$

$\lambda = 1$ eigenvalue, eigenvectors:

$$\begin{pmatrix} -0.13 & 0.16 \\ 0.13 & -0.16 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 16 \\ 13 \end{pmatrix}$$

$\underline{x + y = 1}$ $\quad 16t + 13t = 1$

$29t = 1$

$t = 1/29 \longrightarrow \underline{v} = \begin{pmatrix} 16/29 \\ 13/29 \end{pmatrix}$

(equilibrium state)

5.9. $$A = \begin{pmatrix} 0.61 & 0.13 & 0.03 \\ 0.13 & 0.81 & 0.10 \\ 0.26 & 0.06 & 0.87 \end{pmatrix}$$

regular Markov chain ✓

$\lambda = 1$ eigenvalue, eigenvectors:

$$\begin{array}{l} 100 \cdot \\ 100 \cdot \\ 100 \cdot \end{array}\begin{pmatrix} -0.39 & 0.13 & 0.03 \\ 0.13 & -0.19 & 0.10 \\ 0.26 & 0.06 & -0.13 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\downarrow$

$$\begin{pmatrix} -39 & 13 & 3 \\ 13 & -19 & 10 \\ 26 & 6 & -13 \end{pmatrix} \rightarrow \begin{pmatrix} \boxed{13} & -19 & 10 \\ -39 & 13 & 3 \\ 26 & 6 & -13 \end{pmatrix} \begin{array}{l} \\ 3 \\ \cdot 2 \end{array}$$

$$\rightarrow \begin{pmatrix} \boxed{13} & -19 & 10 \\ 0 & \boxed{-44} & 33 \\ 0 & 44 & -33 \end{pmatrix}$$

$$\Leftarrow \frac{57-40}{4} z = \frac{17}{4} z$$

$$13x = 19y - 10z = 19 \cdot \left(\tfrac{3}{4} z\right) - 10z$$

$$-44y + 33z = 0 \Rightarrow y = \frac{33}{44} z = \underline{\frac{3}{4} z}$$

$\underline{z \text{ free}}$

$$13x = \frac{17}{4}z \Rightarrow x = \frac{17}{52}z$$

$$y = \frac{3}{4}z$$

$$z \text{ free}$$

$$\underline{x+y+z=1}$$

$$\frac{17}{52}z + \frac{3}{4}z + z = 1$$

$$\frac{17+39+52}{52}z = 1$$

$$\frac{108}{52}z = 1$$

$$z = \frac{52}{108} = \frac{13}{27}$$

<u>Equilibrium state:</u>

$$x = \frac{17}{52}\cdot\frac{52}{108} = \underline{\frac{17}{108}}$$

$$y = \frac{3}{4}\cdot\frac{13}{27} = \underline{\frac{39}{108}}$$

$$z = \underline{\frac{52}{108}}$$

<u>5.13</u> $A = \begin{pmatrix} t & 1 & 1 \\ 1 & t & 1 \\ 1 & 1 & t \end{pmatrix}$

a) $|A| = t\cdot(t^2-1) - 1\cdot(t-1) + 1\cdot(1-t)$

$= t^3 - t - t + 1 + 1 - t = \underline{t^3 - 3t + 2}$

$\underline{\lambda = t-1:}$ $A - \lambda I = \begin{pmatrix} \textcircled{1} & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $\underline{rk = 1}$

$\boxed{t-(t-1)=1}$

$$\downarrow$$

$$\begin{pmatrix} \textcircled{1} & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

b) $\underline{t=8:}$ $A = \begin{pmatrix} 8 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 8 \end{pmatrix}$ A is <u>symmetric</u>, hence it is diagonalizable.

$\lambda = 7$ is an eigenvalue with 2 free var's from a).
$\Rightarrow$ mult. of $\lambda = 7$ is at least 2 (or even exactly 2)

$$\text{tr } A = 3\cdot 8 = 24 = \underset{7}{\underset{\shortparallel}{\lambda_1}} + \underset{7}{\underset{\shortparallel}{\lambda_2}} + \underset{10}{\underset{\shortparallel}{\lambda_3}}.$$

$$\begin{vmatrix} 8-\lambda & 1 & 1 \\ 1 & 8-\lambda & 1 \\ 1 & 1 & 8-\lambda \end{vmatrix} = 0$$

$$(8-\lambda)\cdot((8-\lambda)^2-1) - 1\cdot(\underline{8-\lambda-1}) + 1\cdot(1-(8-\lambda))$$

$$(\underline{8-\lambda})(\underline{8-\lambda-1})(8-\lambda+1) - 1\cdot(\underline{8-\lambda-1}) - 1\cdot(\underline{8-\lambda-1}) = 0$$

$$(7-\lambda)\cdot\left[(8-\lambda)(9-\lambda) - 2\right] = 0$$

$\underline{\lambda = 7}$ or $\lambda^2 - 17\lambda + 70 = 0$

$$\lambda = \underline{7}, \underline{10}$$

c) Transition matrix: $A = \begin{pmatrix} 0.8 & 0.10 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{pmatrix}$

Equilibrium state:

$\underline{\lambda=1}$: Eigenvectors

$$\begin{matrix} 10\cdot \\ 10\cdot \\ 10\cdot \end{matrix}\begin{pmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{pmatrix} \rightarrow \begin{pmatrix} \textcircled{1} & 1 & -2 \\ +1 & -2 & +1 \\ -2 & 1 & 1 \end{pmatrix} \begin{matrix}]\cdot -1 \\ \\ ]\,2 \end{matrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ \cancel{0} & \cancel{3} & \cancel{-3} \end{pmatrix}$$

$x = -y + 2z = -z + 2z = \underline{z}$

$y = \underline{z}$

$\underline{z}$ free

Eq: $x + y + z = 1$

$3z = 1 \quad z = 1/3$

$x = 1/3$

$y = 1/3$

$z = 1/3$

$\frac{120}{3} = \underline{40 \text{ cars}}$ at airport.

6.1 iii) $f = xy^2 + x^3y - xy$

$f'_x = y^2 + 3x^2y - y = 0$ $\quad y \cdot (y + 3x^2 - 1) = 0$

$f'_y = 2xy + x^3 - x = 0$ $\quad x \cdot (2y + x^2 - 1) = 0$

$x = 0$ $\quad x = 0$ $\quad y = 0$

$y = 0$ $\quad y = 1$ $\quad x^2 = 1$

$y = \underline{1 - 3x^2}$

$2y + x^2 - 1 = 0$

$2(1 - 3x^2) + x^2 - 1 = 0$

$-5x^2 + 1 = 0$

$x^2 = 1/5$

$y = 1 - 3/5 = 2/5$

Stationary pts:

$(0,0)$, $(0,1)$, $(\pm 1, 0)$,

$(\pm\sqrt{1/5}, 2/5)$

$$H(f) = \begin{pmatrix} 6xy & 2y + 3x^2 - 1 \\ 2y + 3x^2 - 1 & 2x \end{pmatrix}$$

$(x,y) = (0,0)$: $\underset{H(f)(0,0)}{\overset{}{H \,\|}} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ indefinite $D_2 = -1 < 0$ $\Rightarrow$ saddle pt.

$(x,y) = (0,1)$: $H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ — " — saddle pt

$(x,y) = (\pm 1, 0)$: $H = \begin{pmatrix} 0 & 2 \\ 2 & \pm 2 \end{pmatrix}$ indefinite $D_2 = -4$ saddle pts

$$(x,y) = \left(\pm\sqrt{1/5}, 2/5\right) \Rightarrow H = \begin{pmatrix} \pm \frac{12}{5}\sqrt{\frac{1}{5}} & \frac{4}{5} + \frac{3}{5} - 1 = \frac{2}{5} \\ 2/5 & \pm 2\sqrt{\frac{1}{5}} \end{pmatrix}$$

$D_1 = \pm \frac{12}{5}\sqrt{\frac{1}{5}}$

$D_2 = \frac{24}{5} \cdot \frac{1}{5} - \frac{4}{25} = \frac{20}{25} > 0$

$(x,y) = (\sqrt{1/5}, 2/5)$: Pos. defn. $\Downarrow$ local min

$(x,y) = (-\sqrt{1/5}, 2/5)$: neg. defn. $\Downarrow$ local max.

6.2 ii) $f = \underbrace{(x^2+2y^2+3z^2)}_{u} e^{\overbrace{-(x^2+y^2+z^2)}^{v}} = u \cdot e^v$

$$f'_x = 2x \cdot e^v + u \cdot e^v \cdot (-2x) = 2x e^v \cdot (1-u) = 0$$
$$f'_y = 4y e^v + u \cdot e^v \cdot (-2y) = 2y e^v (2-u) = 0$$
$$f'_z = 6z e^v + u \cdot e^v \cdot (-2z) = 2z e^v (3-u) = 0$$

$x=0$ or $u=1$
$y=0$ or $u=2$
$z=0$ or $u=3$

$x=y=z=0$
or
$x=0, y=0, u=3$
or
$x=0, u=2, z=0$
or
$u=1, y=0, z=0$

Stationary pts:
$(0,0,0)$, $(0,0,\pm 1)$,
$(0,\pm 1,0)$, $(\pm 1, 0, 0)$

H(f):
$$f''_{xx} = \underline{2} \, \underline{e^v}(1-u) + \underline{2x} \cdot \underline{e^v}(-2x)(1-u) + \underline{2x} \, \underline{e^v} \cdot (-2x)$$
$$= \underline{\underline{2e^v \left[1-u-2x^2(1-u) - 2x^2\right]}} = 2e^v(1-u-2x^2(2-u))$$

$(uvw)' = (uv)' \cdot w$
$+ uv \cdot w'$
$= (u'v+uv')w$
$+ uvw'$
$= u'vw$
$+ uv'w$
$+ uvw'$

$$f''_{xy} = \underline{2x} \cdot \underline{e^v}(-2y) \cdot (1-u) + \underline{2x} \, \underline{e^v}(-2y)$$
$$= -4xy e^v (1-u+1) = \underline{-4xy e^v (2-u)}$$

⋮

$$H = \begin{pmatrix} * & * & \cdot \\ * & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & xy \cdot \overset{0}{\overset{\shortparallel}{*}} & xz \cdot \overset{0}{\overset{\shortparallel}{*}} \\ xy \cdot \underset{0}{\underset{\shortparallel}{*}} & \cdot & yz \cdot \underset{0}{\underset{\shortparallel}{*}} \\ xz \cdot \underset{0}{\underset{\shortparallel}{*}} & yz \cdot \underset{0}{\underset{\shortparallel}{*}} & \cdot \end{pmatrix}$$

$(x,y,z) = (0,0,0)$:

$$H = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$D_1 = 2$
$D_2 = 8$
$D_3 = 48$

pos. defn.
$\Downarrow$
local min

All other pts: $(x,y,z) \neq (0,0,\pm 1)$:

$$H = \begin{pmatrix} \cdot & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & \cdot \end{pmatrix}$$

both pos. and neg. diagonal entries in all other pts except $(0,0,\pm 1)$
$\Downarrow$
indefinite

$(x,y,z) = (0,0,\pm 1)$: local maxima

(not all computations were completed in class...)

Note: Problem 6.8 is relevant for the midterm.