

Test of Prerequisites

Problems

1. Simplify the following expression as much as possible:

$$\frac{1 - 4x + 4x^2}{x - 1/2}$$

2. Find numbers A and B such that

$$\frac{x - 1}{x^2 - 5x + 6} = \frac{A}{x - 2} + \frac{B}{x - 3}$$

3. Compute the following determinant:

$$\begin{vmatrix} x & 4 & 2 \\ -1 & 2 & x \\ 3 & -x & 5 \end{vmatrix}$$

4. Find the derivative of the function $f(x) = x \ln(x^2) + 5$.

5. Find the derivative of the function

$$f(x) = \frac{e^x}{x^2 + 1}$$

6. Find the derivative of the function

$$f(x) = \frac{3x^2 - 27}{3 - 4x + x^2}$$

7. Compute the integral $\int x e^{-x} dx$.

8. Compute the integral $\int x \sqrt{x^2 + 1} dx$.

9. Find the local maxima and minima of the function $f(x) = x^3 - 3x + 7$.

10. Find the partial derivatives of the function $f(x, y) = e^{xy-x-y}$.

11. Find all stationary points of the function $f(x, y) = \ln(x^2 + y^2 + 7)$.

Answers

1 Simplify the following expression as much as possible:

$$\frac{1 - 4x + 4x^2}{x - 1/2} = 4x - 2$$

2 Find numbers A and B such that

$$\frac{x - 1}{x^2 - 5x + 6} = \frac{-1}{x - 2} + \frac{2}{x - 3}$$

3 Compute the determinant following determinant:

$$\begin{vmatrix} x & 4 & 2 \\ -1 & 2 & x \\ 3 & -x & 5 \end{vmatrix} = x^3 + 24x + 8$$

4 The derivative is $f'(x) = \ln(x^2) + 2 = 2\ln(x) + 2$.

5 The derivative is

$$f'(x) = \frac{e^x(x-1)^2}{(x^2+1)^2}$$

6 The derivative is

$$f'(x) = \frac{-12}{(x-1)^2}$$

7 The integral is $\int x e^{-x} dx = -(x+1)e^{-x} + \mathcal{C}$.

8 The integral is $\int x\sqrt{x^2+1} dx = \frac{1}{3}(x^2+1)^{3/2} + \mathcal{C}$.

9 The function has a local maximum at $x = -1$ and a local minimum at $x = 1$.

10 The partial derivatives are $f'_x = (x-1)e^{xy-x-y}$ and $f'_y = (y-1)e^{xy-x-y}$.

11 There is a unique stationary points at $(x, y) = (0, 0)$.