Problem Session 1
GRA 6035 Mathematics

October 2, 2012

BI Norwegian Business School

Problems

1. Compute the determinant and rank of the following matrices:

$$i)\quad A=\begin{pmatrix}3&-4&-8\\-2&1&4\\2&-2&-5\end{pmatrix}\qquad ii)\quad A=\begin{pmatrix}2&-3&0\\-3&2&0\\0&0&9\end{pmatrix}$$

$$iii)\quad A=\begin{pmatrix}4&-2&0\\-2&3&-2\\0&-2&2\end{pmatrix}$$

$$iv)\quad A=\begin{pmatrix}1&t&-2\\2&4&-t\\-t&-4&-4\end{pmatrix}\qquad \text{for}\quad \begin{cases}\text{general } t\\ t=-2\end{cases}$$

$$v)\quad A=\begin{pmatrix}1-a&1&-2a\\1&0&1\\1&a&-a\end{pmatrix}\qquad \text{for}\quad \begin{cases}\text{general } a\\ a=1\end{cases}$$

$$vi)\quad A=\begin{pmatrix}t&t&1\\0&t&0\\1&0&t\end{pmatrix}\qquad \text{for}\quad \begin{cases}\text{general } t\\ t=1\end{cases}$$

$$vii)\quad A=\begin{pmatrix}a-2&2a-4&-1\\1&a-1&2\\2&2&a+2\end{pmatrix}\qquad \text{for}\quad \begin{cases}\text{general } a\\ a=0\end{cases}$$

Use the general value of the parameter in iv) - vii) if you can. If not, use the specific value.

2. Find all eigenvalues and eigenvectors of the matrices in Problem 1. Use the specific value of the parameter in iv) - vii).

3. Determine if the matrices in Problem 1 are diagonalizable. For those matrices that are diagonalizable, find a diagonal matrix D and an invertible matrix P such that $P^{-1}AP=D$. Use the specific value of the parameter in iv) - vii).

4. In each case, check if the vector $\mathbf{v}$ is an eigenvector for the matrix A. If so, what is the eigenvalue?

$$a)\quad \mathbf{v}=\begin{pmatrix}-2\\-2\\0\end{pmatrix},\ A=\begin{pmatrix}3&-4&-8\\-2&1&4\\2&-2&-5\end{pmatrix}\qquad b)\quad \mathbf{v}=\begin{pmatrix}-1\\2\\-2\end{pmatrix},\ A=\begin{pmatrix}1&t&-2\\2&4&-t\\-t&-4&-4\end{pmatrix}$$

5. Are the following vectors linearly independent?

$$\mathbf{v}_1=\begin{pmatrix}1\\3\\-1\\0\end{pmatrix},\quad \mathbf{v}_2=\begin{pmatrix}-2\\2\\0\\1\end{pmatrix},\quad \mathbf{v}_3=\begin{pmatrix}-1\\0\\1\\1\end{pmatrix},\quad \mathbf{v}_4=\begin{pmatrix}1\\-1\\2\\-1\end{pmatrix}$$

6. Assume that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly independent vectors. Show that the vectors

$$3\mathbf{a} - 2\mathbf{b} + \mathbf{c}, 2\mathbf{a} + \mathbf{b}, \mathbf{a} + 2\mathbf{b}$$

are linearly independent.

7. For which values of a are the following vectors linearly independent?

$$\mathbf{v}_1 = \begin{pmatrix} -1 \\ a \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ a \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ -1 \end{pmatrix}$$

8. Let $Q(x,y,z) = ax^2 + 4xy + y^2 + 2xz + 3z^2$. For what values of a is the quadratic form positive definite?