Mock Exam:	**GRA 60353**	**Mathematics**	
Examination date:	December 2012		Total no. of pages: 2
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus		
Answer sheets:	Squares		
	Counts 80% of GRA 6035		The subquestions are weighted equally
			Responsible department: Economics

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$$

(a) Compute the determinant and rank of A.
(b) Compute all eigenvalues of A. Is A diagonalizable?

Question 2.

We consider the function f with parameter h, given by $f(x, y; h) = hx^4 + y^4 + 4x^2 - (6+h)xy + 4y^2 - 3h$. The function f is defined for all points $(x, y) \in \mathbb{R}^2$.

(a) Compute the Hessian matrix of f, and show that f is convex when $h = 0$. Then determine all values of h such that f is convex.
(b) Find the global minimum of f when $h = 0$.
(c) Will the global minimum value $f^*(h)$ increase or decrease when the value of the parameter h changes from $h = 0$ to a small positive value?

Question 3.

Solve the following difference and differential equations:

(a) $y_{t+2} - 5y_{t+1} + 4y_t = 2^t$
(b) $y' = t(y-1)^2, \quad y(0) = 3$
(c) $(2y - e^t)y' = ye^t + 2e^{2t}, \quad y(0) = 2$

Question 4.

We consider the optimization problem

$$\max\ x + 2y + 2z \text{ subject to } \begin{cases} x^2 + y^2 + z^2 \leq 4 \\ x \geq 0 \\ y \geq 0 \\ z \geq 0 \end{cases}$$

Sketch the set of admissible points, and solve the optimization problem.

Question 5.

Let $a, b \in \mathbb{R}$ be parameters with $a \neq 0$, and consider the matrix A given by

$$A = \begin{pmatrix} b & a & a & a \\ a & b & a & a \\ a & a & b & a \\ a & a & a & b \end{pmatrix}$$

Show that $\lambda = b - a$ is an eigenvalue of A, and find its multiplicity. Use this to find $\det(A)$.