NORWEGIAN BUSINESS SCHOOL

Solutions:	**GRA 60352**	**Mathematics**		
Examination date:	12.10.2012	14:00 – 15:00	Total no. of pages:	2
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions are weighted equally	
Ordinary exam			Responsible department: Economics	

Correct answers: A-A-C-B-C-C-C-X

Question 1.

We reduce the augmented matrix to echelon form:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 & 1 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -2 & 1 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 7 \end{array}\right)$$

From the pivot positions, we see that the system is inconsistent. The correct answer is alternative **A**.

Question 2.

We form the matrix A with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant:

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 4 \\ 1 & 3 & 0 \end{vmatrix} = 1(0-12) + 1(0-2) = -12 - 2 = -14 \neq 0$$

Therefore, the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent. Hence the correct answer is alternative **A**.

Question 3.

We reduce the matrix A to an echelon form:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 6 & 6 & t \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -6 & t-6 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & t-6 \end{pmatrix}$$

We see that the rank of A is three if $t \neq 6$, and two if $t = 6$. The correct answer is alternative **C**.

Question 4.

The characteristic equation of A is $\lambda^2 - 5\lambda + 4 = 0$, and therefore that it has eigenvalues $\lambda = 4$ and $\lambda = 1$. The correct answer is alternative **B**.

Question 5.

The eigenvalues of A are $\lambda = 1$ (with multiplicity two) and $\lambda = 3$, since we have

$$\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & s+1 & s \\ 0 & 1-\lambda & 4 \\ 0 & 0 & 3-\lambda \end{pmatrix} = (1-\lambda)^2(3-\lambda) = 0$$

We compute the eigenvectors of $\lambda = 1$, the eigenvalue of multiplicity 2, by reducing the matrix $A - I$ to an echelon form:

$$\begin{pmatrix} 0 & s+1 & s \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & s+1 & s \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & s+1 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

We see that there are two degrees of freedom for $s = -1$, and one degree of freedom for $s \neq -1$. Therefore, A is diagonalizable for $s = -1$ and not diagonalizable otherwise. The correct answer is alternative **C**.

Question 6.

The symmetric matrix of the quadratic form $Q(x_1, x_2, x_3) = x_1^2 - 4x_1x_2 + 2x_2^2 - 3x_3^2$ is

$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

The leading principal minors are $D_1 = 1$, $D_2 = -2$ and $D_3 = 6$. Since $D_2 < 0$, A is indefinite, and the correct answer is alternative **C**.

Question 7.

We compute the Hessian matrix of $f(x, y, z) = 2x^2 + hy^3 + 3z^4$: First, we compute the first order partial derivatives

$$f'_x = 4x,\ f'_y = 3hy^2,\ f'_z = 12z^3$$

and then we compute the second order partial derivatives and form the Hessian matrix

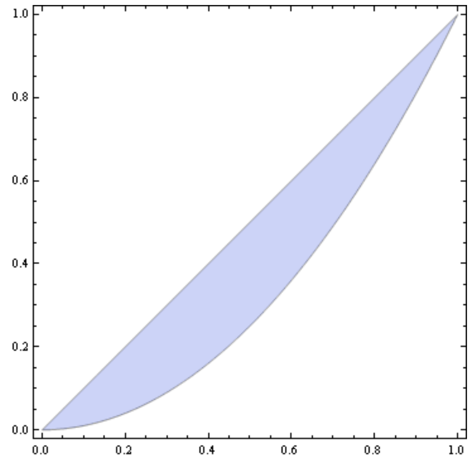
$$H(f) = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 6hy & 0 \\ 0 & 0 & 36z^2 \end{pmatrix}$$

The principal minors of order one are all equal to $4, 6hy, 36z^2$. If $h \neq 0$, then $6hy$ can be both positiv and negative, and it follows that f is not convex for $h \neq 0$. If $h = 0$, then all principal minors $\Delta_k \geq 0$, hence f is convex. The correct answer is alternative **C**.

QUESTION 8.

The problem stated: Consider the subset $S = \{(x, y) : x \le y \le x^2 \text{ and } 0 \le x \le 1\}$ of $\mathbb{R}^2$, the region bounded by the graphs of $y = x^2$ and $y = x$ on $0 \le x \le 1$. This was a misprint, the inequality was supposed to be $x^2 \le y \le x$ and not $x \le y \le x^2$. When $0 < x < 1$, the graph of $y = x$ lies over the graph of $y = x^2$.

Using the inequalities $x^2 \le y \le x$ that were intended, or by interpreting the text *the region bounded by the graphs of* $y = x^2$ *and* $y = x$ *on* $0 \le x \le 1$, we would end up with the region showed in the figure below.



In this case, the set S would be closed, bounded and convex, and the correct answer would be alternative **A**.

On the other hand, using the inequalities $x \le y \le x^2$ as they were printed in the problem, one would end up with the region consisting only of the end-points $(0, 0)$ and $(1, 1)$, since $x \le y \le x^2$ has no solutions when $0 < x < 1$. In this case, S would be closed and bounded, but not convex, and the correct answer would be alternative **B**.