Written examination:	**GRA 60353** **Mathematics**		
Examination date:	13.12.2012	09:00 – 12:00	Total no. of pages: 2
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus		
Answer sheets:	Squares		
	Counts 80% of GRA 6035	The subquestions are weighted equally	
		Responsible department: Economics	

QUESTION 1.

We consider the matrix A given by

$$A = \begin{pmatrix} t & 1 & 1 \\ t & 2 & 1 \\ 4 & t & 2 \end{pmatrix}$$

(a) Compute the determinant and rank of A.
(b) Compute all eigenvalues of A when $t = -2$. Is A diagonalizable when $t = -2$?

QUESTION 2.

We consider the function f with parameter h, given by $f(x, y, z; h) = 12 - x^4 - hx^2 - 3y^2 + 6xz - 6z^2 + h^2$. The function f is defined for all points $(x, y, z) \in \mathbb{R}^3$.

(a) Compute the Hessian matrix of f, and show that f is concave if and only if $h \geq H$ for a constant H. What is the value of H?
(b) Find the global maximum point $(x^*(h), y^*(h), z^*(h))$ of f when $h \geq H$.
(c) Will the global maximum value $f^*(h)$ increase or decrease when the value of the parameter h increases? We assume that the initial value of h satisfies $h \geq H$.

QUESTION 3.

Find the general solution of the following differential equations:

(a) $y'' - 5y' + 6y = 10e^{-t}$
(b) $4te^{2t}y - (1 - 2t)e^{2t}y' = 0$ (when $t > 1/2$)

QUESTION 4.

We consider a model for housing prices, where p_t is the price after t years. The model is given by the difference equation

$$p_{t+2} - 2p_{t+1} + p_t = -15, \quad p_0 = 695, \quad p_1 = 743$$

(a) Solve the difference equation.
(b) We define $d_t = p_{t+1} - p_t$ to be the change in housing prices. Show that $d_{t+1} - d_t$ is constant, and use this to determine when housing prices will increase and when housing prices will decrease.

QUESTION 5.

We consider the following optimization problem:

$$\max \ln(x^2 y) - x - y \text{ subject to } \begin{cases} x + y \geq 4 \\ x \geq 1 \\ y \geq 1 \end{cases}$$

Sketch the set of admissible points, and solve the optimization problem.