BI NORWEGIAN SCHOOL OF MANAGEMENT

Department of Economics

Solutions:	**GRA 60352 Mathematics**
Examination date:	24.05.2011, 09:00 – 10:00
Permitted examination aids:	Bilingual dictionary. BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus^{TM}
Answer sheets:	Answer sheet for multiple choice examinations
Total number of pages:	2

Correct answers: A-D-B-D-A-C-C-B

Question 1.

Since the augmented matrix of the system is in echelon form, we see that the system is inconsistent. Hence the correct answer is alternative $\boxed{A}$.

Question 2.

We compute the determinant

$$\begin{vmatrix} 2 & 1 & h+1 \\ 3 & 2 & h \\ -1 & 1 & h-2 \end{vmatrix} = 3h+3$$

Hence the vectors are linearly independent exactly when $h \neq -1$, and the correct answer is alternative $\boxed{D}$. This question can also be answered using Gauss elimination.

Question 3.

We compute an echelon form of A using elementary row operations, and get

$$A = \begin{pmatrix} 2 & 10 & 6 & 8 \\ 1 & 5 & 4 & 11 \\ 3 & 15 & 7 & -2 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 5 & 4 & 11 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence A has rank 2, and the correct answer is alternative $\boxed{B}$. This question can also be answered using minors.

Question 4.

The characteristic equation of A is $\lambda^2 - 9\lambda + 20 = 0$. Hence the eigenvalues of A is $\lambda = 4, \lambda = 5$, and the correct answer is alternative $\boxed{D}$.

Question 5.

In order for $\mathbf{v}$ to be an eigenvector, we must have $A\mathbf{v} = \lambda\mathbf{v}$, or $2 + b = \lambda \cdot 1$ and $-1 + 3b = \lambda b$. This gives $\lambda = b + 2$ and $b^2 - b + 1 = 0$, and there is no solution for b. Hence the correct answer is alternative $\boxed{A}$.

Question 6.

The symmetric matrix associated with Q is $A = \left(\begin{smallmatrix} -2 & 6 \\ 6 & 2 \end{smallmatrix}\right)$, and we compute its eigenvalues to be $\pm\sqrt{40}$. Hence the correct answer is alternative $\boxed{C}$.

Question 7.

The function f is a sum of a linear function and a quadratic form with symmetric matrix

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Since A has eigenvalues $\lambda = -2 \pm \sqrt{2}$ and $\lambda = -1$, the quadratic form is negative definite and therefore concave (but not convex). Hence the correct answer is alternative $\boxed{C}$.

Question 8.

We compute $A^2 = I$ directly, and use this to show that $A^7 = (A^2)^3 \cdot A = A$. The correct answer is therefore alternative $\boxed{B}$. Alternatively, we may compute that $\lambda = \pm 1$ are eigenvalues of A and find corresponding eigenvectors $\mathbf{v}_1 = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$ and $\mathbf{v}_2 = \left(\begin{smallmatrix} -1 \\ 1 \end{smallmatrix}\right)$. This gives $D = P^{-1}AP$ with

$$D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad P^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

and therefore $A^7 = (PDP^{-1})^7 = PD^7P^{-1} = PDP^{-1} = A = \left(\begin{smallmatrix} 1 & 2 \\ 0 & -1 \end{smallmatrix}\right)$.