

This is a test, which can be taken before the FORK 1003 Preparatory course in Mathematics (to see if you need to follow the course), and after the preparatory course (to see if you have learned the material and are prepared for your MSc courses).

A score of 50p or more is acceptable, a score of 65p or more is very good, and a score of 75-80p or more is excellent. The max score is 84p.

**Question 1.**

We consider the matrices  $A$  and  $B$  given by

$$A = \begin{pmatrix} 1 & 4 & -1 & 5 & 7 \\ 3 & -1 & 2 & 2 & 4 \\ 5 & 7 & 0 & 12 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 & 7 \\ 3 & -1 & 4 \\ 5 & 7 & 8 \end{pmatrix}$$

- a) **(6p)** Solve the linear system  $A\mathbf{x} = \mathbf{0}$ . How many free variables are there?
- b) **(6p)** Compute  $|B|$ , and find  $B^{-1}$  if it exists.
- c) **(6p)** Compute  $A^2$ ,  $AB$ ,  $BA$ ,  $B^2$  when the product is defined.

**Question 2.**

We consider the function  $f(x) = \frac{2}{3} \ln(1+x) + \frac{1}{3} \ln(1-x)$ .

- a) **(6p)** Compute  $f'(x)$  and find the stationary points of  $f$ .
- b) **(6p)** Compute  $f''(x)$  and determine when  $f$  is convex and/or concave.
- c) **(6p)** Show that  $f$  has a maximum, and find the maximum value of  $f$ .

**Question 3.**

Compute the indefinite integrals:

- a) **(6p)**  $\int \frac{2-x}{x^2} dx$
- b) **(6p)**  $\int x \ln x dx$
- c) **(6p)**  $\int x \sqrt{x^2+3} dx$
- d) **(6p)**  $\int \frac{6}{4-x^2} dx$

**Question 4.**

We consider the function  $f(x, y) = x^3 - 3xy + y^3$ .

- a) **(6p)** Find the partial derivatives and stationary points of  $f$ .
- b) **(6p)** Classify the stationary points of  $f$ .
- c) **(12p)** Show that the Lagrange problem has a maximum, and find its maximum value:

$$\max x + y \text{ when } x^3 - 3xy + y^3 = 0$$