

FORK1003

Solutions for Exercises 3

August 3, 2016

1 Introduction to Determinants

Solution 1.1.

- (a) 12 (b) 26 (c) 0

2 Clever Trick for 3×3 Determinants

Solution 2.1.

- (a) 174 (b) 0 (c) -24

3 Cofactor Expansion

3.1 Minors and Cofactors

Solution 3.1.

(a) $\begin{bmatrix} 0 & 3 & 1 \\ 2 & 0 & 6 \\ -2 & -5 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -5 & 4 \\ -1 & 2 & 6 \\ 3 & -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 0 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & -5 \end{bmatrix}$

Solution 3.2.

- (a) -8 (c) 30
(b) 26 (d) 4

3.2 Cofactor Expansion

Solution 3.3.

- (a) 30 (c) 30 (3rd column)
 (b) 25 (d) -496 (4th row)

4 Determinants by Row Reduction

4.1 Determinants and Elementary Row Operations

Solution 4.1.

- (a) $-|A|$ (d) $|A|$
 (b) $-2|A|$ (e) $2|A|$
 (c) $|A|$ (f) $2|A|$

Solution 4.2.

- (a) -78 . Possible sequence of row operations:

$$\begin{aligned} R1 &\rightarrow \frac{1}{2}R1 \\ R2 &\rightarrow R2 - R1 \\ R3 &\leftrightarrow R2 \\ R3 &\rightarrow R3 + 2R2 \end{aligned}$$

- (b) 13. Possible sequence of row operations:

$$\begin{aligned} R1 &\leftrightarrow R2 \\ R1 &\rightarrow -R1 \\ R2 &\rightarrow R2 + 5R1 \\ R3 &\rightarrow R3 - 2R1 \\ R3 &\rightarrow R3 + R2 \end{aligned}$$

- (c) 256. Possible sequence of row operations:

$$\begin{aligned} R2 &\rightarrow R2 - R1 \\ R3 &\rightarrow R3 - 2R1 \\ R4 &\rightarrow R4 - 6R1 \\ R3 &\rightarrow R3 + R2 \\ R4 &\rightarrow R4 + 8R2 \\ R4 &\rightarrow R4 - \frac{5}{4}R4 \end{aligned}$$

(d) -20 . Possible sequence of row operations:

$$\begin{aligned}
 R1 &\leftrightarrow R4 \\
 R2 &\leftrightarrow R3 \\
 R3 &\rightarrow R3 - 4R1 \\
 R4 &\rightarrow R4 - 3R1 \\
 R3 &\rightarrow R3 - R4 \\
 R4 &\rightarrow R4 + 10R2 \\
 R3 &\rightarrow -\frac{1}{5}R3 \\
 R4 &\rightarrow R4 - 12R3
 \end{aligned}$$

Solution 4.3.

(a) -72 . Start by expanding 3rd row.

(b) 336 . Start by the row operation $R5 \rightarrow R5 + R1$, followed by expanding the 1st column.

Solution 4.4.

$$(a) \det(A) = 9. \quad A^{-1} = \frac{1}{9} \begin{bmatrix} 2 & -3 & -2 \\ 4 & 12 & 5 \\ 3 & 9 & 6 \end{bmatrix}$$

$$(b) \det(B) = -23. \quad A^{-1} = -\frac{1}{23} \begin{bmatrix} -4 & 3 & -1 \\ 2 & -13 & 12 \\ 3 & -8 & 18 \end{bmatrix}$$

Solution 4.5.

(a)

$$|A| = -9, |A_1| = 6, |A_2| = -3, |A_3| = 0.$$

$$(x_1, x_2, x_3) = (-2/3, 1/3, 0)$$

(b)

$$|A| = -38, |A_1| = 38, |A_2| = -38, |A_3| = -38.$$

$$(x_1, x_2, x_3) = (-1, 1, 1)$$