

FORK1003

Exercises for Lecture 1

August 1, 2016

1 Introduction to Linear Systems

1.1 Linear Equation

Exercise 1.1. Are these equations linear or nonlinear?

(a) $2x_1 + 2x_2 - 3x_3 = 9$

(e) $x_1 - 2x_2^{1/3} = 2$

(b) $2x_1x_2x_3 = 0$

(f) $3(x_1 + x_2) - 2(x_3 - x_4) = 3$

(c) $3x_1^2 - 3x_2 = 3$

(g) $(x_1 + x_2)(x_3 - x_4) = -5$

(d) $x_1 - 2^{1/3}x_2 = 2$

2 Solutions of Linear Systems

Exercise 2.1. Solve these linear systems by substitution:

(a)
$$\begin{cases} x_1 + 2x_2 = 10 \\ -2x_1 + 3x_2 = 1 \end{cases}$$

(b)
$$\begin{cases} -x_1 - x_2 = -2 \\ 5x_1 + 3x_2 = 5 \end{cases}$$

(c)
$$\begin{cases} x_1 - x_2 + 3x_3 = 5 \\ 4x_2 - 3x_3 = -8 \\ -x_2 + 4x_3 = 2 \end{cases}$$

(d)
$$\begin{cases} x_1 - 5x_2 - x_3 = 14 \\ 2x_1 - x_3 = 0 \\ -x_1 + 3x_2 = -10 \end{cases}$$

Exercise 2.2. How many solutions do these linear systems have?

$$(a) \begin{cases} 16x_1 - 4x_2 = 8 \\ -2x_1 + \frac{1}{2}x_2 = -1 \end{cases}$$

$$(c) \begin{cases} x_1 - x_2 = 10 \\ x_1 + 3x_2 = 14 \end{cases}$$

$$(b) \begin{cases} 3x_1 - 2x_2 = 4 \\ 9x_1 - 6x_2 = -2 \end{cases}$$

3 Row Reduction

3.1 Coefficient & Augmented Matrix

Exercise 3.1. Write out the coefficient matrices of the following linear systems:

$$(a) \begin{cases} 3x_1 + 2x_2 = -3 \\ x_1 - x_2 + x_3 = 0 \\ -2x_1 - 3x_2 + 2x_3 = 4 \end{cases}$$

$$(b) \begin{cases} x_1 - x_3 = 2 \\ x_2 + 3x_3 = -1 \\ -4x_1 + 10x_2 - x_3 = 0 \\ x_1 + x_3 = 0 \end{cases}$$

$$(c) \begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 6 \\ x_2 - 10x_3 + 8x_4 - \frac{1}{2}x_5 = -2 \end{cases}$$

Exercise 3.2. Write out the augmented matrices of the following linear systems:

$$(a) \begin{cases} x_1 - 3x_2 + 8x_3 - x_4 = 1 \\ x_3 - 8x_4 = 13/3 \\ -2x_1 - x_2 + 3x_3 = 0 \end{cases}$$

$$(b) \begin{cases} 6x_1 = 8 \\ 3x_2 = -4 \\ -4x_3 = 2 \\ 18x_4 = 4 \end{cases}$$

$$(c) \begin{cases} 2x_1 - 7x_2 - 6x_3 - x_4 = 16 \\ x_2 + 11x_3 - \frac{3}{2}x_4 - \frac{1}{2}x_5 = 2 \end{cases}$$

Exercise 3.3. Express these augmented matrices as linear systems:

$$(a) \left[\begin{array}{ccc|c} 2 & 3 & 4 & 5 \\ 1 & -2 & -3 & 6 \end{array} \right]$$

$$(b) \left[\begin{array}{cc|c} -2 & 0 & 10 \\ 13 & 2 & -16 \\ -3 & 4 & 0 \\ 4 & 2 & 3 \end{array} \right]$$

3.2 Elementary Row Operations

Exercise 3.4. Apply the given row operation to the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 6 \\ 0 & 3 & 5 & 7 \\ -2 & 4 & 0 & 13 \\ 16 & -6 & 7 & -1 \\ 0 & -1 & 0 & 2 \end{array} \right].$$

(a) $R3 \leftrightarrow R5$

(d) $R1 \rightarrow R1 + R2$

(b) $R2 \rightarrow R2 - 3R1$

(e) $R5 \rightarrow \frac{1}{2}R5$

(c) $R4 \rightarrow -2R4$

Exercise 3.5. Solve the following linear system using augmented matrices and elementary row operations:

$$\begin{cases} 2x_1 - x_2 = 4 \\ 3x_1 + 2x_2 = 13 \end{cases}$$

Exercise 3.6. Solve the following linear system using augmented matrices and elementary row operations:

$$\begin{cases} 3x_2 - 3x_3 = 9 \\ 2x_1 - x_3 = -7 \\ 3x_1 + 2x_2 + x_3 = 0 \end{cases}$$

3.4 Echelon Forms

Exercise 3.7. Determine whether each matrix is in echelon form, and if so, whether it is in reduced echelon form.

(a)
$$\begin{bmatrix} 3 & 0 & 2 & 0 & 4 & 6 \\ 0 & -1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 4 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & 2 & 4 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3.5 Pivot Positions & Basic Variables

Exercise 3.8. For each echelon form matrix, give the pivot positions, pivot columns, basic variables and free variables. Find a solution set to the linear system.

$$(a) \left[\begin{array}{cccc|c} 1 & -2 & 0 & 6 & 2 \\ 0 & 1 & -1 & 3 & 2 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$(b) \left[\begin{array}{cccc|c} 1 & 3 & -2 & 4 & 2 \\ 0 & 1 & 0 & -2 & 10 \\ 0 & 0 & 1 & 7 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

Exercise 3.9. For each linear system, reduce the augmented matrix to reduced echelon form, give the pivot positions, pivot columns, basic variables and free variables.

$$(a) \begin{cases} x_1 + 2x_2 - x_3 + x_4 = 1 \\ 3x_1 - x_2 + 2x_3 - x_4 = 3 \\ -x_2 - x_3 + x_4 = 1 \\ -x_1 + 4x_2 - 5x_3 + 4x_4 = 0 \end{cases}$$

$$(b) \begin{cases} -x_1 + x_2 + 3x_3 - x_4 = 0 \\ -3x_1 - 2x_2 - 2x_3 = -3 \\ 7x_1 + 3x_2 + x_3 + x_4 = 6 \\ 2x_1 + 3x_2 + 5x_3 - x_4 = 3 \end{cases}$$