Solutions to Lecture 1: Matrices and Matrix Algebra

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Problem 6. Compute $A + B$ and $5A$ when

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix}.$$

Solution.

$$A + B = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -2 & 1 \end{pmatrix}$$

$$5A = 5 \cdot \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 15 \\ 5 & 5 \end{pmatrix}$$

Problem 7. Compute $A + B$, $A - B$ and $3A - 2B$ when

$$A = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix}.$$

Solution.

$$A + B = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ -1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 5 & 0 \\ 0 & 0 \end{pmatrix}$$

$$3A - 2B = 3 \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 12 & 0 \\ 0 & 1 \end{pmatrix}$$

Problem 8. Compute AB and BA when

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix}.$$

Solution.

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix} = \begin{pmatrix} -13 & -6 \\ -5 & -3 \end{pmatrix}$$

$$BA = \begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -7 & -9 \\ -6 & -9 \end{pmatrix}$$

Problem 9. Compute AB and BA, if possible, for the following:

(1) $A = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(2) $A = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$

(3) $A = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix}$

Solution.

(1)

$$AB = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 2 & 3 \\ 6 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 9 \\ 0 & 1 \end{pmatrix}$$

(2)

$$AB = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 3 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 2 & 0 \\ 6 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 8$$

(3)

AB is not defined.

$$BA = \begin{pmatrix} 2 & 3 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 4 & 6 \\ 1 & 1 \end{pmatrix}$$

Problem 10. The percentage that will vote for parties Left, Center and Right is given as follows:

	Left	Center	Right	No. of voters
Oslo	46 %	12 %	42 %	550 000
Akershus	40 %	12 %	48 %	500 000
Vestfold	46 %	10 %	44 %	253 000

Use matrix multiplication to compute the total number of voters for each party in the three regions.

Solution.

$$\tfrac{1}{100} \begin{pmatrix} 46 & 40 & 46 \\ 12 & 12 & 10 \\ 42 & 48 & 44 \end{pmatrix} \begin{pmatrix} 550000 \\ 500000 \\ 253000 \end{pmatrix} = \begin{pmatrix} 569\,380 \\ 151\,300 \\ 582\,320 \end{pmatrix}$$

Left gets 389 380, Center gets 151 300 and Right gets 366 320.