FORK1003 Linear algebra
Short review of Lecture 1

Eivind Eriksen

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Example

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 \\ 1 & 0 \\ 6 & 5 \end{pmatrix} \qquad C = \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}$$

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We can calculate the sum of A and C

$$A + C = \begin{pmatrix} 2 & 1 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 2+4 & 1+(-1) \\ 3+2 & 6+0 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 5 & 6 \end{pmatrix},$$

but the sum of A and B is not defined, since they do not have the same order.

Example

If

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 6 \end{pmatrix}$$

we get that

$$4A = \begin{pmatrix} 4 \cdot 2 & 4 \cdot 1 \\ 4 \cdot 3 & 4 \cdot 6 \end{pmatrix} = \begin{pmatrix} 8 & 4 \\ 12 & 24 \end{pmatrix}.$$

Definition

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Example

The following are examples of zero matrices:

$$\underset{2\times 2}{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}} \quad \underset{2\times 4}{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}} \quad \underset{3\times 1}{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

Definition

The square matrix of order $n \times n$ that have only 1's on the diagonal and 0's elsewhere, is called the identity matrix and is denoted by I or I_n.

1.1 Matrices and Matrix Addition

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Example

$$I_2 = \underset{2\times 2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \text{ and } I_3 = \underset{3\times 3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

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The following is *not* an identity matrix, since it is not a square matrix.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Definition

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Example

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix} \implies A^T = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{pmatrix}.$$

Proposition

Let A, B and C be matrices of the same size (order), and let r and s be scalars (numbers). Then:

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + 0 = A$
4. $r(A + B) = rA + rB$
5. $(r + s)A = rA + sA$
6. $r(sA) = (rs)A$

Example

Let $A = \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Compute AB.

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Solution

	$\begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$
$\begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 \cdot 2 + 3 \cdot 1 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$

Example

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Solution

$$\begin{array}{c|c} & \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 5 & 1 \cdot 0 + 3 \cdot 1 & * \\ * & * & * \\ * & * & * \end{pmatrix} \end{array}$$

1.2 Matrix Multiplication

Example

Let $A = \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Compute AB.

Solution

$$\begin{array}{c|c} & \begin{pmatrix} 2 & 0 & \color{red}{1} \\ 1 & 1 & \color{red}{0} \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ \color{red}{0} & \color{red}{1} \end{pmatrix} & \begin{pmatrix} 5 & 3 & 1 \\ -6 & 0 & -3 \\ 1 & 1 & \color{red}{0 \cdot 1 + 1 \cdot 0} \end{pmatrix} \end{array}$$

Solution

$$\begin{array}{c|c} & \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 3 \\ -3 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 5 & 3 & 1 \\ -6 & 0 & -3 \\ 1 & 1 & 0 \cdot 1 + 1 \cdot 0 \end{pmatrix} \end{array}$$

$$AB = \begin{pmatrix} 5 & 3 & 1 \\ -6 & 0 & -3 \\ 1 & 1 & 0 \end{pmatrix}.$$

Note the orders of the matrices:

$$\underset{3\times 2}{A}\ \underset{2\times 3}{B} = \underset{3\times 3}{AB}.$$

Proposition

We have the following rules for matrix multiplication:

1. $(AB)C = A(BC)$ (associative law)
2. $A(B + C) = AB + AC$ (left distributive law)
3. $(A + B)C = AC + BC$ (right distributive law)
4. $IA = AI = A$
5. $(AB)^T = B^T A^T$