Exercise session problems

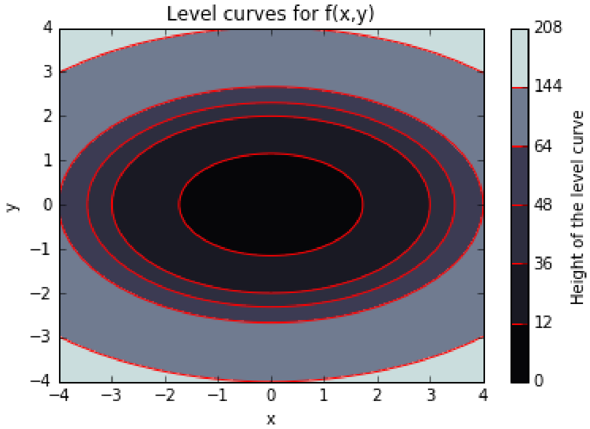
Problem 1.
Consider the subset $D \subseteq \mathbb{R}^2$ given by the inequality $y(x-2) \leq 3$. Make a sketch of $D = \{(x,y) : y(x-2) \leq 3\}$, and mark the inner points and the boundary points of D. Is D compact?

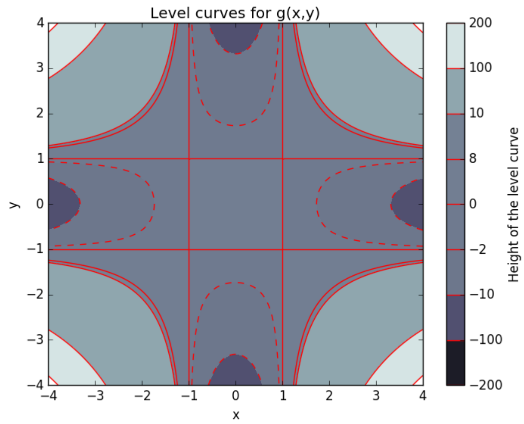
Problem 2.
Consider a subset of the plane $\mathbb{R}^2$ given by the following conditions. Determine whether the subset is compact. It is useful to make a sketch of the area.

a) $2x + 3y = 6$
b) $2x + 3y < 6$
c) $2x + 3y \leq 6$
d) $x^2 + y^2 = 4$
e) $x^2 + y^2 \geq 4$
f) $x^2 + y^2 \leq 4$
g) $x^2 - 2x + 4y^2 = 4$
h) $x^2 - 2x + 4y^2 \leq 4$
i) $x^2 - 2x + 4y^2 \geq 4$
j) $xy = 1$
k) $xy \leq 1$
l) $xy \geq 1$
m) $\sqrt{x^2 + y^2} = 3$
n) $\sqrt{x^2 + y^2} \leq 3$
o) $x^2y^2 - x^2 - y^2 + 1 = 0$
p) $x^2y^2 - x^2 - y^2 + 1 = 1$

Problem 3.
State the extreme value theorem. Give examples of a set D in the plane which is closed, but not bounded, as well as a set E in the plane which is bounded, but not closed. Can you find a function $f(x,y)$ which does not have a maximum nor a minimum in D, and a function which does not have a maximum nor a minimum in E?

Problem 4.
Level curves for the functions $f(x,y) = 4x^2 + 9y^2$ and $g(x,y) = x^2y^2 - x^2 - y^2 + 1$ in the area $-4 \leq x,y \leq 4$ are shown in the figures below.





a) Find max / min $f(x,y)$ when $-4 \leq x,y \leq 4$ by using the figure.
b) Find max / min $g(x,y)$ when $-4 \leq x,y \leq 4$ by using the figure.
c) Find max / min $f(x,y)$ when $x^2 + y^2 = 16$ by using the figure.
d) Find max / min $g(x,y)$ when $x = y$ by using the figure.

Problem 5.
Solve the optimization problems:

a) $\max / \min f(x,y) = x^3 - 3xy + y^3$ when $0 \le x,y \le 1$
b) $\max / \min f(x,y) = x^3 - 3xy + y^3$ when $0 \le x,y \le 2$
c) $\max / \min f(x,y) = e^{xy-x-y}$ when $0 \le x,y \le 2$
d) $\max / \min f(x,y) = xy(x^2 - y^2)$ when $-1 \le x,y \le 1$
e) $\max / \min f(x,y) = (x^2 - 1)(y^2 - 1)$ when $-1 \le x,y \le 1$

Problem 6.
Find the maximum- and minimum value for the optimization problem

$$\max / \min f(x,y) = \sqrt{xy} - x \text{ when } 0 \le x,y \le 1$$

Review problems

Problem 7.
In a labor market, let e_t be the share of those employed and u_t be the share of those unemployed after t months. We assume that the change over the course of one month is that 4% of those employed become unemployed, and 12% of those unemployed find work.

a) Write down the equations that express e_1 and u_1 using e_0 and u_0, both written out and expressed in matrix form.
b) What proportion are unemployed after 2 months if $u_0 = 0.15$?
c) We say that the market is in equilibrium after t months if $e_{t+1} = e_t$ and $u_{t+1} = u_t$. What proportion is unemployed if the market is in equilibrium?
d) Use Wolfram Alpha, Excel, or other tools to calculate e_t and u_t for $t = 60$ (after five years) if $u_0 = 0.15$. Compare your answer with the equilibrium you found above.

Problem 8.
Compute the derivative of the function $f(x)$:

a) $f(x) = \int_0^x 2ze^{-z^2}\,dz$
b) $f(x) = \int_0^x e^{-z^2}\,dz$
c) $f(x) = \int_0^{\sqrt{x}} 2ze^{-z^2}\,dz$

Optional: Exercises from the Norwegian textbook

Textbook [E]:	Eriksen, *Matematikk for økonomi og finans*
Exercise book [O]:	Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Exercises:	[E] 7.6.1 - 7.6.2
Solution manual:	See [O] Ch. 7.6

Answers to the exercise session problems

Problem 1.
Boundary points are given by the equation $y(x-2)=3$, that is points on the graph of $y=3/(x-2)$ (a hyperbola). Inner points are given by $y(x-2)<3$, that is points under the hyperbola when $x>2$, and points over the hyperbola when $x<2$, as well as all points where $x=2$. The set D is not compact (closed, but not bounded).

Problem 2.

a) No	b) No	c) No	d) Yes	e) No	f) Yes	g) Yes	h) Yes
i) No	j) No	k) No	l) No	m) Yes	n) Yes	o) No	p) No

Problem 4.

a) $f_{\text{min}}=0$ in (0,0), and $f_{\text{max}}=208$ in $(\pm 4,\pm 4)$

b) $f_{\text{min}}=-15$ in $(0,\pm 4)$ and $(\pm 4,0)$, and $f_{\text{max}}=225$ in $(\pm 4,\pm 4)$

c) $f_{\text{min}}=64$ in $(\pm 4,0)$, and $f_{\text{max}}=144$ in $(0,\pm 4)$

d) $f_{\text{min}}=0$ in (1,1) and $(-1,-1)$, and $f_{\text{max}}=225$ in (4,4) and $(-4,-4)$

Problem 5.

a) $f_{\text{max}}=1,\ f_{\text{min}}=-1$

b) $f_{\text{max}}=8,\ f_{\text{min}}=-1$

c) $f_{\text{max}}=1,\ f_{\text{min}}=1/e^2$

d) $f_{\text{max}}=2\sqrt{3}/9,\ f_{\text{min}}=-2\sqrt{3}/9$

e) $f_{\text{max}}=1,\ f_{\text{min}}=0$

Problem 6.
See the final exam of MET11807 06/2021 Exercise 5: $f_{\text{max}}=1/4,\ f_{\text{min}}=-1$

Problem 7.

a) $e_1=0.96e_0+0.12u_0,\ u_1=0.04e_0+0.88u_0$ or $\mathbf{v}_1=A\mathbf{v}_0$ with $A=\begin{pmatrix}0.96 & 0.12\\ 0.04 & 0.88\end{pmatrix}$, $\mathbf{v}_t=\begin{pmatrix}e_t\\ u_t\end{pmatrix}$

b) $e_2\approx 0.82,\ u_2\approx 0.18$

c) $0.25=25\%$

d) $e_{60}\approx 0.750003, u_{60}\approx 0.249997$; Wolfram Alpha: { {0.96,0.12}, {0.04,0.88} }^(60)*{{0.85},{0.15}}

Problem 8.

a) $2xe^{-x^2}$

b) e^{-x^2}

c) e^{-x}