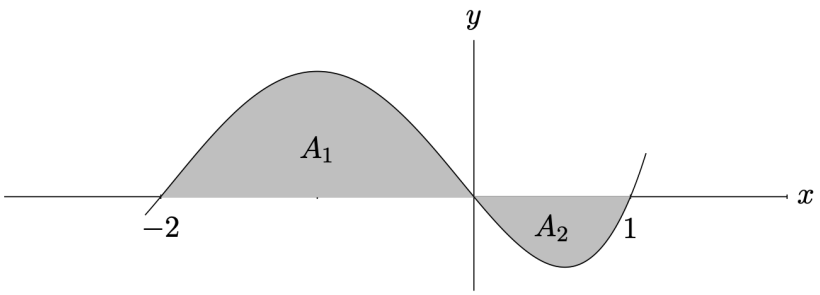
Exercise session problems

Integration

Problem 1.
Compute the following integrals:

a) **(6p)** $\int 30x\sqrt{x}\ \mathrm{d}x$ b) **(6p)** $\int xe^{-x}\ \mathrm{d}x$ c) **(6p)** $\int \frac{6-3x}{4-9x^2}\ \mathrm{d}x$



d) The graph of a function f is shown in the figure above. Determine the area A_1 when you are given that $A_2 = 22/15$ and

$$\int_{-2}^{1} f(x)dx = \frac{18}{5}.$$

Problem 2.
We purchase a property in order to rent it out. We expect to receive net rental income as a continuous cash flow, in such a way that the cash flow after t years is given by $I(t) = 12e^{0.07t}$ (in million NOK per year). We use continuous discounting when computing the net present value (NPV), with discount rate $r = 10\%$.

a) Find the NPV of the cash flow we receive if the property is rented out forever.

b) We will consider to sell the property after 7 years if the NPV of the sales price is at least equal to the NPV of the future rental income. What must the sales price be in order for us to consider selling?

Problem 3.

Compute the integrals. Show the integration rules you use.

a) $\int x(1-x)^2 \, \mathrm{d}x$ b) $\int \frac{x}{1-x^2} \, \mathrm{d}x$ c) $\int \frac{x}{(1-\sqrt{x})^2} \, \mathrm{d}x$

The function f is defined for $-4 \leq x \leq 3$ and has the graph shown in Figure 3.

d) Which value of a gives the greatest value for the definite integral $\int_{-4}^{a} f(x) \, \mathrm{d}x$?

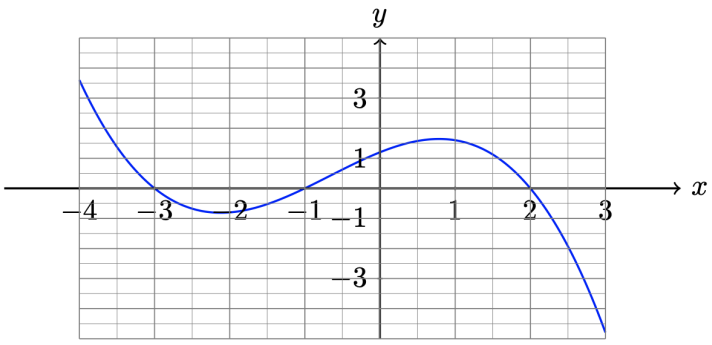


FIGURE 3. The graph of $y = f(x)$

Problem 4.

Compute these indefinite integrals. Write down which integration methods you use.

(a) **(6p)** $\int \frac{2}{\sqrt{x}} \, \mathrm{d}x$

(b) **(6p)** $\int \frac{12}{4-x^2} \, \mathrm{d}x$

(c) **(6p)** $\int 9\sqrt{x}\ln(\sqrt{x}) \, \mathrm{d}x$

Linear algebra

Problem 5.

We consider a linear system $A\mathbf{x} = \mathbf{b}$ with parameter a, where

$$A = \begin{pmatrix} 2 & 9 & 5 & -9 \\ 4 & a & 10 & -18 \\ 1 & 5 & 2 & -2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 7 \\ 22 \\ 3 \end{pmatrix}$$

(a) **(6p)** Solve the linear system when $a = 22$.
(b) **(6p)** Determine the values of a such that the linear system is consistent.

Problem 6.

We consider a linear system with parameter a, given in matrix form as

$$\begin{pmatrix} 2 & -6 & 4 & 6 \\ 3 & a & 7 & 2 \\ 1 & -2 & 1 & 10 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ 5 \end{pmatrix}$$

(a) **(6p)** Solve the linear system when $a = -12$.
(b) **(6p)** Determine the values of a (if any) such that the linear system has no solutions.

Problem 7.

Let the matrix A be given by

$$A = \begin{pmatrix} a & 2 & 3 \\ 2 & a & 3 \\ 2 & 3 & a \end{pmatrix}$$

(a) **(6p)** Find A^{-1} when $a = 1$.
(b) **(6p)** Compute the determinant $|A|$ for an arbitrary value of a, and determine when $|A| = 0$.

Optimization

Problem 8.

We consider the function f given by $f(x,y) = x^2y - 5xy^2 + xy^3$.

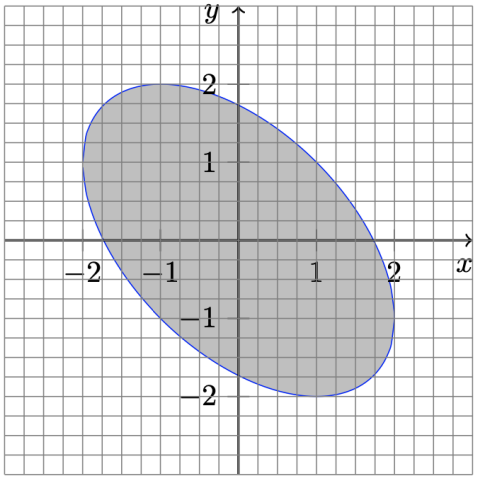
(a) **(6p)** Find the three stationary points of f.
(b) **(6p)** Compute the Hessian of f, and classify the stationary points $(x,y) \neq (0,0)$.

Problem 9.

In the figure below, the blue curve is given by the equation $g(x,y) = a$, and the marked region is given by the inequality $g(x,y) \leq a$. We consider the maximum problem

$$\max f(x,y) = x + y \text{ when } g(x,y) \leq a$$

(a) **(6p)** Show that the maximum problem has a solution that lies on the blue curve.
(b) **Extra credit (6p)** Use the figure to estimate the maximum value. Give reasons for your answer.



Answers to the exercise session problems

Problem 1.
Exam MET1180 F19 Problem 3.

Problem 2.
Exam MET1180 S19 Problem 4.

Problem 3.
Exam EBA1180 S20 Problem 4.

Problem 4.
Exam EBA1180 F2021 Problem 2.

Problem 5.
Exam EBA1180 F22 Problem 1.

Problem 6.
Exam EBA1180 S22 Problem 1.

Problem 7.
Exam EBA1180 F22 Problem 3.

Problem 8.
Exam EBA1180 S22 Problem 4.

Problem 9.
Exam EBA1180 F21 Problem 5.