Problem Sheet 3
DRE 7007 Mathematics

BI Norwegian Business School

Problems

1. We consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is continuous at $x = 0$. Is f differentiable at $x = 0$? Is f a C^1 function? What about the function $g : \mathbb{R} \to \mathbb{R}$ given by

$$g(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

2. For each of the functions, compute the Hessian matrix at the given point:

a) $f(x,y) = x^2 + \sqrt{y}$ at $(x,y) = (1,1)$
b) $f(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$ at $(x,y,z) = (2,2,2)$
c) $f(x,y,z) = xy + yz + xz$ at $(x,y,z) = (1,1,1)$

3. Determine the definiteness of the Hessian matrices in Problem 2.

4. For each of the functions, compute the Hessian matrix at a general point:

a) $f(x,y) = e^{xy} - 1$
b) $f(x,y,z) = xyz$

5. Classify the points where f is positive (semi)definite, negative (semi)definite and indefinite for each of the functions in Problem 4.

6 (Difficult). We consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Show that f is a C^1 function, and compute its Hessian matrix. Is it a C^2 function?

Keep answers as short and to the point as possible. Answers must be justified.