Problem Sheet 1
DRE 7007 Mathematics

BI Norwegian Business School

Problems

1. Compute all eigenvalues and eigenvectors for the following matrices:

$$(a)\quad A = \begin{pmatrix} 2 & -3 \\ 7 & -8 \end{pmatrix} \qquad (b)\quad A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 0 & 0 \\ 1 & -1 & 2 \end{pmatrix} \qquad (c)\quad A = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$$

2. For each of the matrices in Exercise 1, use the eigenvalues and eigenvectors to answer the following questions:

a) Compute $\det(A)$ and $\mathrm{rk}(A)$.
b) Determine if A is positive (semi)definite, negative (semi)definite or indefinite.
c) Is A diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

3. Determine if the matrices are positive (semi)definite, negative (semi)definite or indefinite:

$$(a)\quad \begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 5 \\ 0 & 5 & 8 \end{pmatrix} \qquad (b)\quad \begin{pmatrix} 1 & -2 & -1 & 1 \\ -2 & 1 & 1 & 2 \\ -1 & 1 & -1 & -3 \\ 1 & 2 & -3 & 0 \end{pmatrix}$$

4. Write the following dynamical system (in discrete time) in matrix form:

$$\begin{aligned} x_{t+1} &= 0.75x_t + 0.35y_t \\ y_{t+1} &= 0.25x_t + 0.65y_t \end{aligned}$$

We assume that the initial state (x_0, y_0) satisfies $x_0 + y_0 = 1$. Does the system tend towards an equlibrium in the long run (as $t \to \infty$)? If so, what is the equilibrium state?

Keep answers as short and to the point as possible. Answers must be justified.