Exercise Problems

Problem 1.
Consider the sequence given by $x_n = \frac{n^2+1}{n}$. Does the sequence have a limit? Is it bounded?

Problem 2.
Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with the Euclidean norm. Show that the Cauchy-Schwarz inequality implies that the triangle inequality

$$\|\mathbf{x}+\mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$$

holds for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.

Problem 3.
Let $I = [0,1]$, and consider the vector space $C(I,\mathbb{R})$ of continuous functions on I. Show that

$$f \cdot g = \int_0^1 f(t)g(t)\,\mathrm{d}t$$

defines an inner product on $C(I,\mathbb{R})$, and compute $f \cdot g$ when $f(t) = t^2$ and $g(t) = t^3$. What is $d(f,g)$ when d is the metric induced by this inner product?

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