Abstract of Thesis

Eivind Eriksen: *Graded D-modules on Monomial Curves*

Let k be an algebraically closed field of characteristic 0, and let X be an algebraic variety over k. In this thesis, we study several types of *differential structures* on X when we allow X to have singularities.

A. Differential operators. Let $X = \operatorname{Spec} A$ be an affine monomial curve defined over k. In this case, we study the ring D of differential operators on X, as well as the associated graded ring $\operatorname{gr} D$ and the module of derivations $\operatorname{Der}_k(A)$. We give a concrete description of each of these objects in terms of the semigroup that defines X. This part of the thesis will appear in *Journal of Algebra*, and is joint work with Henrik Vosegaard.

B. Modules with connections. We study the notion of a $\mathbf{g}$-connection on an A-module M, when A is a commutative k-algebra and $\mathbf{g} \subseteq \operatorname{Der}_k(A)$ is a Lie algebroid, and give the general obstruction theory for such objects. We use this to study categories of modules with (integrable) $\mathbf{g}$-connections, which we consider to be a natural generalization of the category of modules with (integrable) connections. We show that if $X = \operatorname{Spec} A$ is an affine monomial curve which is Gorenstein, then any graded, torsion free A-module M of rank 1 allows an integrable $\mathbf{g}$-connection with $\mathbf{g} = \operatorname{Der}_k(A)$. In contrast, we show that the only such A-module M which allows a graded D-module structure is $M = A$ with the natural action of D.

C. Algebraic D-modules. Let D be the first Weyl algebra defined over k, and consider the category of graded, holonomic D-modules. We give a complete classification of this category by finding all the indecomposable objects. This gives a complete classification of the graded, holonomic D'-modules when D' is the ring of differential operators on any affine monomial curve, via Morita equivalence. The classification is similar to the classification of the local analytic D-modules which are holonomic with regular singularities at a smooth point of dimension 1, obtained by Boutet de Monvel.

D. Iterated extensions. In order to obtain the classification in C, we used the method of *iterated extensions*. This is a general method for studying finite length categories, and it is implicit in the thesis how to use this method to identify and classify all *uniserial* finite length categories. This includes the classification in C, the classification due to Boutet de Monvel mentioned above, and others in the literature. We mention that the preprint *Iterated extensions in module categories*, Eivind Eriksen, Preprint 18/2002 University of Warwick includes further developments in this direction.

E. Noncommutative deformations of modules. This is a generalization of the local or formal deformation theory, and it is due to Laudal. It describes the simultaneous deformation of a finite family of modules over an associative k-algebra. It has applications to noncommutative geometry, in particular moduli problems in algebraic geometry, and it is also essential for the method of iterated extensions mentioned above. For this reason, and because it is not covered in the literature, I have included an account of this deformation theory in the thesis.

F. Hilbert functions. Let A be a positively graded k-algebra with $A_0 = k$, but not necessarily generated in degree 1. We develop the theory of Hilbert functions for finitely generated, graded A-modules M, based on work by Geramita and others. In particular, we show that in this case, there exists a Hilbert quasi-polynomial and natural notions of dimension and multiplicity. We use this to define *dimension* and *multiplicity* for algebraic D-modules over an affine monomial curve.