Free resolutions and the Hilbert scheme of space curves

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Abstract Let $H(d,g)$ be the Hilbert scheme of space curves C with homogeneous ideal $I(C) := H^0_*(\mathcal{I}_C)$ and Rao module $M := H^1_*(\mathcal{I}_C)$. For non-arithmetically Cohen-Macaulay curves C (i.e. $M \neq 0$) we recall a useful theorem of Rao on the form of a free resolution of $I(C)$, as well as Peskine-Szpiro's result concerning how to find a free resolution of a CI-linked curve. For curves with "small" M the non-vanishing of certain graded Betti numbers of $I(C)$ is related to $H(d,g)$ being singular at (C). To see this connection we deform C to a more general curve C' in various ways by e.g. making consecutive free summands in a minimal free resolution of $I(C)$ disappear in a free resolution of $I(C')$ (i.e. "killing" ghost terms). Using this for Buchsbaum curves of diameter one ($M_v \neq 0$ for only one v), we establish a one-to-one correspondence between the set S of irreducible components of $H(d,g)$ that contain (C) and a set of minimal 5-tuples of non-negative integers that specialize in an explicit manner to a 5-tuple of graded Betti numbers of C related to ghost terms. Moreover we completely determine the graded Betti numbers of all generic curves of S, and we give a specific description of the singular locus of the Hilbert scheme of curves of diameter at most one in terms of closures of Betti strata. We also prove some semi-continuity results for the graded Betti numbers of any space curve under some assumptions.